MATHEMATICS
KARNATAKA CET - 2023

Version: **A-1**

KEY ANSWERS

1	** B	16	B	31	A	46	C
2	A	17	C	32	B	47	A
3	C	18	A,C	33	A	48	B
4	B,C	19	C	34	A	49	D
5	B	20	C	35	D	50	D
6	A	21	B,D	36	C	51	C
7	D	22	B	37	B	52	B
8	D	23	C	38	B	53	B
9	D	24	D	39	B	54	D
10	B	25	B	40	B	55	C
11	D	26	C	41	*C	56	*D
12	B	27	A	42	C	57	C
13	C	28	D	43	C	58	B
14	B	29	C	44	C	59	* D
15	B	30	C	45	C	60	D

*** : Grace : Ambiguous or questions with none of the given alternatives as correct answer**
****: out of syllabus**

1. n^{th} term of the series $1+\frac{3}{7}+\frac{5}{7^2}+\frac{1}{7^2}+.......$ is

A) $\frac{2n-1}{7^n}$ B) $\frac{2n-1}{7^{n-1}}$ C) $\frac{2n+1}{7^{n-1}}$ D) $\frac{2n+1}{7^n}$ **Ans. (B)**

Solution : Put n = 1

Then $t_n = t_1 = 1$

Go to the alternatives and put n = 1

(A) : $\frac{1}{7}$ (B) : $\frac{1}{1}=1$ (C) : $\frac{3}{1}=3$ (D) : $\frac{3}{7}$

$\therefore$ (B) is the correct answer

Remark: As AGP is not included in the syllabus, grace marks could be given.

2. If $p\left(\frac{1}{q}+\frac{1}{r}\right)$, $q\left(\frac{1}{r}+\frac{1}{p}\right)$, $r\left(\frac{1}{p}+\frac{1}{q}\right)$ are in A.P., then p, q, r

A) are in A.P. B) are not in A.P. C) are not in G.P. D) are in G.P. **Ans. (A)**

Solution : Add $1=\frac{p}{p}=\frac{q}{q}=\frac{r}{n}$ in the terms and divide by $\left(\frac{1}{p}+\frac{1}{q}+\frac{1}{r}\right)$ to get p, q, r are in A.P.

Note: $p\left(\frac{1}{q}+\frac{1}{r}\right)+1=p\left(\frac{1}{q}+\frac{1}{r}+\frac{1}{p}\right)$; similarly for other terms.

3. A line passes through (2, 2) and its perpendicular to the line 3x + y = 3. Its y-intercept is

A) 1 B) $\frac{1}{3}$ C) $\frac{4}{3}$ D) $\frac{2}{3}$ **Ans. (C)**

Solution : Line : $x - 3y = 2 - 6$

$\therefore$ y-intercept = $\frac{4}{3}$

Remark: It is a repeated CET 15 question.

4. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is

A) $2x^2 - 3y^2 = 7$ B) $x^2 - y^2 = 32$ C) $y^2 - x^2 = 32$ D) $\frac{x^2}{4}-\frac{y^2}{9}=1$ **Ans. (B & C)**

Solution : $2ae = 16$; $e=\sqrt{2}$

$\therefore$ $a = 4\sqrt{2}$; $b^2 = a^2e^2 - a^2 = 64 - 32 = 32$

$\therefore\ \frac{x^2}{32}-\frac{y^2}{32}=1$ or $\frac{y^2}{32}-\frac{x^2}{32}=1$

Remark: It is a repeated CET 18 question

5. If $\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}$ = A cos B, then the values of A and B respectively are

A) 2, 1 B) 2, 2 C) 1, 1 D) 1, 2 **Ans. (B)**

Solution : A cos B = $\lim_{x\to 0}\frac{\cos(2+x)+\cos(2+x)}{1}=2\cos 2\Rightarrow A=2=B$

Aliter: Apply sin C – sin D formula

6. If n is even and the middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ is $924x^6$, then n is equal to

A) 12 B) 10 C) 8 D) 14 **Ans. (A)**

Solution : Let n = 2m

Then middle term = $T_{m+1}={}^{2m}C_m\,(x^2)^m\cdot\left(\frac{1}{x}\right)^m={}^{2m}C_m\cdot x^m$

$\therefore x^m=x^6\Rightarrow m=6\quad\therefore n=12$

7. The mean of 100 observations is 50 and their standard deviation is 5. Then the sum of squares of all observations is

A) 250000 B) 50000 C) 255000 D) 252500 **Ans. (D)**

Solution : $\sigma=\sqrt{\frac{\Sigma x^2}{n}-(\bar{x})^2}\Rightarrow 5=\sqrt{\frac{\Sigma x^2}{100}-2500}$

$\therefore\ \frac{\Sigma x^2}{100}=2525\quad\therefore\ \Sigma x^2=252500$

Remark: Almost the same CET 2019 question.

8. f : R → R and g : [0, ∞) → R are defined by $f(x)=x^2$ and $g(x)=\sqrt{x}$. Which one of the following is not true ?

A) (fog) (2) = 2 B) (gof) (4) = 4 C) (gof) (– 2) = 2 D) (fog) (– 4) = 4 **Ans. (D)**

Solution : (fog) (– 4) = f(g(– 4)) ; but g(– 4) = $\sqrt{-4}$ doesn't exist

Remark: Repeated CET 2019 question.

9. Let f : R → R be defined by $f(x)=3x^2-5$ and g : R → R by $g(x)=\frac{x}{x^2+1}$, then gof is

A) $\frac{3x^2}{x^4+2x^2-4}$ B) $\frac{3x^2-5}{9x^4-30x^2+26}$ C) $\frac{3x^2}{9x^4+30x^2-2}$ D) $\frac{3x^2-5}{9x^4-6x^2+26}$ **Ans. (B)**

Solution : $(gof)(x)=g(f(x))=\frac{f}{f^2+1}=\frac{3x^2-5}{(3x^2-5)^2+1}=\frac{3x^2-5}{9x^4-30x^2+26}$

10. Let the relation R be defined in N by aRb if 3a + 2b = 27 then R is

A) {(1, 12), (3, 9), (5, 6), (7, 3), (9, 0)} B) {(1, 12), (3, 9), (5, 6), (7, 3)}

C) {(2, 1), (9, 3), (6, 5), (3, 7)} D) $\left\{\left(0,\frac{27}{2}\right),(1,12),(3,9),(5,6),(7,3)\right\}$ **Ans. (B)**

Remark: Repeated CET 22 questions

11. Let f(x) = sin 2x + cos 2x and $g(x)=x^2-1$, then g(f(x)) is invertible in the domain

A) $x\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ B) $x\in\left[\frac{-\pi}{4},\frac{\pi}{4}\right]$ C) $x\in\left[0,\frac{\pi}{4}\right]$ D) $x\in\left[\frac{-\pi}{8},\frac{\pi}{8}\right]$ **Ans. (D)**

Solution : $g(f(x))=(\sin 2x+\cos 2x)^2-1=2\sin 2x\cdot\cos 2x=\sin 4x$;

It is invertible if $4x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ i.e., $x\in\left[-\frac{\pi}{8},\frac{\pi}{8}\right]$

Remark: sin x is bijective and hence invertible in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

12. The contrapositive of the statement "If two lines do not intersect in the same plane then they are parallel". Is

A) If two lines are not parallel then they do not intersect in the same plane

B) If two lines are not parallel then they intersect in the same plane

C) If two lines are parallel then they do not intersect in the same plane

D) If two lines are parallel then they intersect in the same plane **Ans. (B)**

Solution : Contrapositive of $p \to q$ is $\sim q \to \sim p$

13. The value of $\cot^{-1}\left[\dfrac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$ where $x \in \left(0, \dfrac{\pi}{4}\right)$ is

A) $x-\dfrac{x}{3}$ B) $\dfrac{x}{2}$ C) $\pi-\dfrac{x}{2}$ D) $\dfrac{x}{2}-\pi$ **Ans. (C)**

Solution : $y=\cot^{-1}\left(\dfrac{\cos\frac{x}{2}-\sin\frac{x}{2}+\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}-\cos\frac{x}{2}-\sin\frac{x}{2}}\right) = \cot^{-1}\left(-\cot\frac{x}{2}\right) = \pi-\cot^{-1}\cot\frac{x}{2}=\pi-\frac{x}{2}$

14. If $x\begin{bmatrix}3\\2\end{bmatrix} + y\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}15\\5\end{bmatrix}$ then the value of x and y are

A) $x=-4, y=-3$ B) $x=4, y=3$ C) $x=-4, y=3$ D) $x=4, y=-3$ **Ans. (B)**

Solution : $3x + y = 15$; $2x - y = 5 \Rightarrow x = 4, y = 3$

15. If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2 =$

A) AB B) A + B C) 2 BA D) 2 AB **Ans. (B)**

Solution : $A^2 + B^2 = AA + BB = A(BA) + B(AB) = (AB)A + (BA)B = BA + AB = A + B$

16. If $A=\begin{bmatrix}2-k & 2\\ 1 & 3-k\end{bmatrix}$ is singular matrix, then the value of $5k - k^2$ is equal to

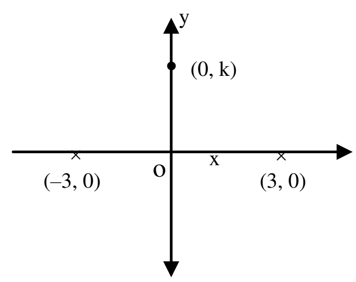
A) – 4 B) 4 C) 6 D) – 6 **Ans. (B)**

Solution : $|A| = (2-k)(3-k) - 2 = 0 \Rightarrow k^2 - 5k + 4 = 0 \quad \therefore 5k - k^2 = 4$

17. The area of a triangle with vertices (– 3, 0), (3, 0) and (0, k) is 9 sq. units, the value of k is

A) 6 B) 9 C) 3 D) – 9 **Ans. (C)**

Solution : $\Delta = \dfrac{1}{2}(6)(k) = 9 \Rightarrow k = 3$



Aliter: $\begin{vmatrix}-3 & 0 & 1\\ 3 & 0 & 1\\ 0 & k & 1\end{vmatrix} = \pm 18 \Rightarrow -k(-6) = \pm 18 \quad \therefore k = 3 \ (\text{or } -3)$

18. If $\Delta=\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$ and $\Delta_1 = \begin{vmatrix}1 & 1 & 1\\ bc & ca & ab\\ a & b & c\end{vmatrix}$, then

A) $\Delta_1 \neq \Delta$ B) $\Delta_1 = \Delta$ C) $\Delta_1 = -\Delta$ D) $\Delta_1 = 3\Delta$ **Ans. (A & C)**

Solution : $\Delta_1 = \dfrac{1}{abc}\begin{vmatrix}a & b & c\\ abc & abc & abc\\ a^2 & b^2 & c^2\end{vmatrix} = \begin{vmatrix}a & b & c\\ 1 & 1 & 1\\ a^2 & b^2 & c^2\end{vmatrix} = -\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{vmatrix} = -\Delta$

Aliter: $\Delta = (a-b)(b-c)(c-a)$

$\Delta_1 = -(a-b)(b-c)(c-a) = -\Delta$

$\therefore \Delta_1 = -\Delta$ and hence $\Delta_1 \neq \Delta$

$\therefore$ (A) and (C) are correct answers

19. If $\sin^{-1}\left(\dfrac{2a}{1+a^2}\right) + \cos^{2}\left(\dfrac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ where $a, x \in (0, 1)$ then the value of x is

A) $\dfrac{2a}{1+a^2}$ B) 0 C) $\dfrac{2a}{1-a^2}$ D) $\dfrac{a}{2}$ **Ans. (C)**

Solution : Given: $4\tan^{-1} a = 2\tan^{-1} x \Rightarrow x = \dfrac{2a}{1-a^2}$

Remark: $\sin^{-1}\frac{2a}{1+a^2} = \cos^{-1}\frac{1-a^2}{1+x^2} = 2\tan^{-1}a$; $\tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x$

20. If $u=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{du}{dv}$ is

A) $\frac{1-x^2}{1+x^2}$ B) $\frac{1}{2}$ C) 1 D) 2 **Ans. (C)**

Solution : Required is derivative of 2 $\tan^{-1}x$ w.r.t. 2 $\tan^{-1}x$ and hence it is 1

21. The function f(x) = cot x is discontinuous on every point of the set

A) $\left\{x=(2n+1)\frac{\pi}{2};\ n\in Z\right\}$ B) $\{x = n\pi;\ n \in Z\}$

C) $\left\{x=\frac{n\pi}{2};\ n\in Z\right\}$ D) $\{x = 2n\pi;\ n \in Z\}$ **Ans. (B & D)**

Solution : cot x is continuous at every multiple of π

(B) is the correct answer

But the set in **(D)** is a subset of the set in **(B)**

i.e. **(B)** is also a correct answer

22. If the function is $f(x) = \frac{1}{x+2}$, then the point of discontinuity of the composite function y = f(f(x)) is

A) $\frac{2}{5}$ B) $\frac{-5}{2}$ C) $\frac{1}{2}$ D) $\frac{5}{2}$ **Ans. (B)**

Solution : $f(f(x))=\frac{1}{\frac{1}{x+2}+2} = \frac{x+2}{2x+5}$ is discontinuous at $x=\frac{-5}{2}$

23. If y = a sin x + b cos x, then $y^2+\left(\frac{dy}{dx}\right)^2$ is a

A) function of x and y B) function of x C) constant D) function of y **Ans. (C)**

Solution : y = a sin x + b cos x ; y′ = a cos x – b sin x

$\therefore y^2 + (y')^2 = (a \sin x + b \cos x)^2 + (a \cos x - b \sin x)^2 = 2(a^2 + b^2)$, a constant

24. If $f(x) = 1 + nx + \frac{n(n-)}{2}x^2+\frac{n(n-1)(n-2)}{6}x^3+........+x^n$, then $f''(1) =$

A) $n(n-1)2^n$ B) $(n-1)2^{n-1}$ C) 2^{n-1} D) $n(n-1)2^{n-2}$ **Ans. (D)**

Solution : $f(x) = (1 + x)^n$

$f'(x) = n(1 + x)^{n-1}$; $f''(x) = n(n-1)(1 + x)^{n-2}$

$\therefore f''(1) = n(n-1)\, 2^{n-2}$

Remark: It is a repeated CET 09 question

25. If $A=\begin{bmatrix}1 & \tan\alpha/2\\ -\tan\alpha/2 & 1\end{bmatrix}$ and AB = I then B =

A) $\cos^2 \alpha/2 . 1$ B) $\cos^2 \alpha/2 . A^T$ C) $\sin^2 \alpha/2 . A$ D) $\cos^2 \alpha/2 . A$ **Ans. (B)**

Solution : $B = A^{-1} = \frac{1}{\sec^2\frac{\alpha}{2}}\begin{bmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{bmatrix} = \cos^2\frac{\alpha}{2}A'$

26. A circular plate of radius 5 cm is heated. Due to expansion, its radius increases at the 0.05 cm/sec. The rate at which its area is increasing when radius is 5.2 cm is

A) 5.05 π cm^2/sec B) 5.2 π cm^2/sec C) 0.52 π cm^2/sec D) 24.4 π cm^2/sec **Ans. (C)**

Solution : $A = \pi r^2$

$\frac{dA}{dt}=2\pi r\frac{dr}{dt}$ $\therefore \left.\frac{dA}{dt}\right|_{r=5.2} = 2\pi\times5.2\times0.05=0.52\,\pi$

27. The distance 's' in meters travelled by a particle in 't' seconds is given by $s=\frac{2t^3}{3}-18t+\frac{5}{3}$. The acceleration when the particle comes to rest is

A) 12 m^2/sec B) 3 m^2 /sec C) 18 m^2 / sec D) 10 m^2 /sec **Ans. (A)**

Solution : $\frac{ds}{dt}=2t^2-18$; $\frac{ds}{dt}=0 \Rightarrow t=3$

$\frac{d^2s}{dt^2}=4t$ $\therefore$ Required = 12 m^2/s

28. A particle moves along the curve $\frac{x^2}{16}+\frac{y^2}{4}=1$. When the rate of change of abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is

A) III or IV B) I or III C) II or III D) II or IV **Ans. (D)**

Solution : $\frac{2x\frac{dx}{dt}}{16}=-\frac{2y\frac{dy}{dt}}{4}$; Given $\frac{dx}{dt}=4\frac{dy}{dt} \Rightarrow \frac{x\,.\,\frac{4\,dy}{dt}}{16}=\frac{-y\frac{dy}{dt}}{4} \Rightarrow x=-y$

As x and y are of opposite signs, (D) is the correct answer.

29. An enemy fighter jet is flying along the curve given by $y = x^2 + 2$. A soldier is placed at (3, 2) wants to shoot down the jet when it is nearest to him. Then the nearest distance is

A) 2 units B) $\sqrt{3}$ units C) $\sqrt{5}$ units D) $\sqrt{6}$ units **Ans. (C)**

Solution : z = Distance between (3, 2) and $(x, x^2 + 2)$ = $\sqrt{(x-3)^2+x^4}$

z is min $\Leftrightarrow$ z^2 is min : let $f(x) = z^2 = (x-3)^2 + x^4$

$f'(x) = 2(x-3) + 4x^3 \Rightarrow f'(1) = 0$

$\therefore$ z is min at x = 1 and min z = $\sqrt{4+1}=\sqrt{5}$

30. $\int_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}}+5^{\sqrt{10-x}}}\,dx =$

A) 4 B) 5 C) 3 D) 6 **Ans. (C)**

Solution : $I=\frac{1}{2}(8-2)=3$. From memory !

Remark: $\int_a^b \frac{f(a+b-x)}{f(x)+f(a+b-x)}\,dx = \frac{1}{2}(b-a)$

31. $\int \sqrt{\text{cosec } x-\sin x}\;\; dx =$

A) $2\sqrt{\sin x}+C$ B) $\sqrt{\sin x}+C$ C) $\frac{2}{\sqrt{\sin x}}+C$ D) $\frac{\sqrt{\sin x}}{2}+C$ **Ans. (A)**

Solution : $I=\int\sqrt{\frac{1-\sin^2 x}{\sin x}}\,dx=\int\frac{\cos x}{\sqrt{\sin x}}\,dx=\int\frac{d\,(\sin x)}{\sqrt{\sin x}} = 2\sqrt{\sin x}+C$

32. If f(x) and g(x) are two functions with g(x) = $x-\frac{1}{x}$ and $fog(x)=x^3-\frac{1}{x^3}$, then $f'(x)$ =

A) $x^2-\frac{1}{x^2}$ B) $3x^2 + 3$ C) $1-\frac{1}{x^2}$ D) $3x^2+\frac{3}{x^4}$ **Ans. (B)**

Solution: $f(g(x))=x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)^3+3\left(x-\frac{1}{x}\right) = [g(x)]^3 + 3[g(x)]$ $\therefore$ $f(x) = x^3 + 3x$

$f'(x) = 3x^2 + 3$

33. $\int\frac{1}{1+3\sin^2 x+8\cos^2 x}\,dx =$

A) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$ B) $\frac{1}{6}\tan^{-1}(2\tan x)+C$

C) $6\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

D) $\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

Ans. (A)

Solution : $\int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x + 8}\,dx = \int \frac{\sec^2 x}{3^2 + (2\tan x)^2}\,dx$ $\quad \left| \begin{array}{l} 2\tan x = t \\ \sec^2 x\,dx = \frac{1}{2}dt \end{array}\right.$

$$= \int \frac{\frac{1}{2}dt}{3^2 + t^2} = \frac{1}{2}\times\frac{1}{3}\tan^{-1}\frac{t}{3} = \frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$$

34. $\int_{-2}^{0} (x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1))dx =$

A) 4 B) 0 C) 1 D) 3 **Ans. (A)**

Solution : $\int_{-2}^{0} ((x+1)^3 + 2)\,dx + \int_{-2}^{0} (x+1)\cos(x+1)\,dx$

$$= \left[\frac{(x+1)^4}{4} + 2x\right]_{-2}^{0} + \int_{-1}^{1} t\cos t\,dt = \left(\frac{1}{4}+0\right) - \left(\frac{1}{4}-4\right) + 0 = 4$$

Aliter: Put t = x + 1

$\therefore\ I = \int_{-1}^{1} (t^3 + 2 + t\cos t) = \int_{-1}^{1} 2\,dt = 4$

35. $\int_{0}^{\pi} \frac{x\tan x}{\sec x\,.\,\mathrm{cosec}\,x}\,dx =$

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) $\frac{\pi^2}{2}$ D) $\frac{\pi^2}{4}$ **Ans. (D)**

Solution : $I = \int_{0}^{\pi} \frac{x\tan x}{\sec x\ \mathrm{cosec}\ x}dx = \int_{0}^{\pi} x\sin^2 x\,dx = \int_{0}^{\pi} (\pi - x)\sin^2 x\,dx$

$\therefore\ 2I = \pi\int_{0}^{\pi}\sin^2 x\,dx = \pi\,2\int_{0}^{\pi/2}\sin^2 x\,dx = 2\pi\,.\,\frac{\pi}{4} \quad \therefore\ I = \frac{\pi^2}{4}$

36. $\int\sqrt{5-2x+x^2}\,dx =$

A) $\frac{x-1}{2}\sqrt{5+2x+x^2} + 2\log\left|(x-1)+\sqrt{5+2x+x^2}\right| + C$

B) $\frac{x-1}{2}\sqrt{5-2x+x^2} + 2\log\left|(x+1)+\sqrt{x^2+2x+5}\right| + C$

C) $\frac{x-1}{2}\sqrt{5-2x+x^2} + 2\log\left|(x-1)+\sqrt{5-2x+x^2}\right| + C$

D) $\frac{x}{2}\sqrt{5-2x+x^2} + 4\log\left|(x+1)+\sqrt{x^2-2x+5}\right| + C$ **Ans. (C)**

Solution : $I = \int\sqrt{(x^2-2x+1)+4}\ dx = \int\sqrt{(x-1)^2+2^2}\,dx\,(x-1)$

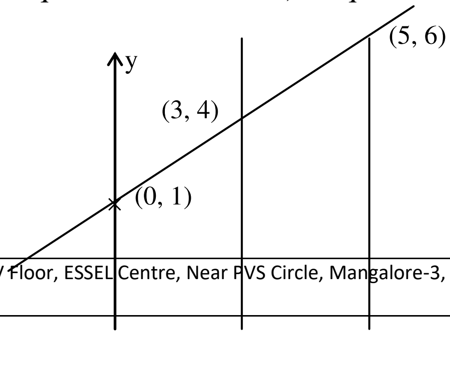
$$I = \frac{(x-1)}{2}\sqrt{(x-1)^2+2^2} + \frac{2^2}{2}\log_e\left|(x-1)+\sqrt{(x-1)^2+4}\right| + C$$

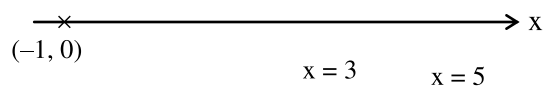
Note: Very poor alternatives and hence the answer can be guessed !

37. The area of the region bounded by the line y = x + 1, and the lines x = 3 and x = 5 is

A) $\frac{11}{2}$ sq. units B) 10 sq. units C) 7 sq. units D) $\frac{7}{2}$ sq. units **Ans. (B)**

Solution :





$$A=\int_3^5 (x+1)\,dx=\left.\frac{x^2}{2}+x\right|_3^5=\left(\frac{25}{2}+5\right)-\left(\frac{9}{2}+3\right)=\frac{35}{2}-\frac{15}{2}=10\text{ sq. units}$$

Aliter: A = area of a trapezium = $\frac{1}{2}(4+6)(5-3)=10$

38. If a curve passes through the [point (1, 1) and at any point (x, y) on the curve, the product of the slope of its tangent and x co-ordinate of the point is equal to the y co-ordinate of the point, then the curve also passes through the point

A) (–1, 2) B) (2, 2) C) $(\sqrt{3}, 0)$ D) (3, 0) **Ans. (B)**

Solution : $\frac{dy}{dx}.x=y\Rightarrow\frac{dy}{y}=\frac{dx}{x}\Rightarrow\log y=\log x+\log c\Rightarrow y=cx$

As (1, 1) lies on it, 1 = c $\therefore$ y = x

39. The degree of the differential equation $1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2=\sqrt[3]{\frac{d^2y}{dx^2}+1}$ is

A) 1 B) 6 C) 2 D) 3 **Ans. (B)**

Solution : $\left(1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2\right)^3=\frac{d^2y}{dx^2}+1$

40. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then

A) $\vec{a}$ and $\vec{b}$ are coincident B) $\vec{a}$ and $\vec{b}$ are perpendicular

C) inclined to each other at 60° D) $\vec{a}$ and $\vec{b}$ are parallel **Ans. (B)**

Solution : $\vec{a}$ and $\vec{b}$ are $\perp$r $\therefore$ diagonals of a parallelogram are equal $\Rightarrow$ rectangle

41. The component of $\hat{i}$ in the direction of the vector $\hat{i}+\hat{j}+2\hat{k}$ is

A) $6\sqrt{6}$ B) $\sqrt{6}$ C) $\frac{\sqrt{6}}{6}$ D) 6 **Ans. (C)**

Solution : The component of $\hat{i}$ in the direction of unit vector along the given vector is $\frac{1}{\sqrt{6}}=\frac{\sqrt{6}}{6}$.

Remark: The wording is unusual or wrong. Hence grace marks can be awarded.

42. In the interval $\left(0, \frac{\pi}{2}\right)$, area lying between the curves y = tan x and y = cot x and the X-axis is

A) 4 log 2 sq. units B) 3 log 2 sq. units C) log 2 sq. units D) 2 log 2 sq. units **Ans. (C)**

Solution : $A=\int_0^{\pi/4}\tan x\,dx+\int_{\pi/4}^{\pi/2}\cot x\,dx=\left.\log\sec x\right|_0^{\pi/4}+\left.\log\sin x\right|_{\pi/4}^{\pi/2}$

$$=\log\sqrt{2}=0+0-\log\frac{1}{\sqrt{2}}=\frac{1}{2}\log 2+\frac{1}{2}\log 2=\log 2$$

43. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ and $(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})=\lambda(\vec{b}\times\vec{c})$, then the value of λ is equal to

A) 4 B) 2 C) 6 D) 3 **Ans. (C)**

Solution : $\vec{a}\times2\vec{b}=2\vec{b}\times3\vec{c}=3\vec{c}\times\vec{a}$ $\therefore$ $\vec{a}\times\vec{b}=3\vec{b}\times\vec{c}$ and $\vec{c}\times\vec{a}=2(\vec{b}\times\vec{c})$

By data , $6(\vec{b}\times\vec{c})=\lambda(\vec{b}\times\vec{c})\Rightarrow\lambda=6$

Remark: $\vec{a}+\vec{b}+\vec{c}=\vec{0}\Rightarrow\vec{a}, \vec{b}, \vec{c}$ represent the sides of triangle in that order and hence $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

44. If a line makes an angle of $\frac{\pi}{3}$ with each X and Y axis then the acute angle made by Z-axis is

A) $\frac{\pi}{2}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$ **Ans. (C)**

Solution : $\frac{1}{4}+\frac{1}{4}+\cos^2\gamma=1 \Rightarrow \cos^2\gamma=\frac{1}{2}$ $\therefore \cos\gamma=\frac{1}{\sqrt{2}}$ $\therefore\ \gamma=\frac{\pi}{4}$

45. The length of perpendicular drawn from the point (3, –1, 11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is

A) $\sqrt{33}$ B) $\sqrt{66}$ C) $\sqrt{53}$ D) $\sqrt{29}$ **Ans. (C)**

Solution : Any point on the line P(2λ, 3λ + 2, 4λ + 3) ; A ≡ (3, –1, 11)

AP is ⊥ the line if (2λ – 3) (2) + (3λ + 3) . 3 + (4λ – 8) . 4 = 0

⇒ 4λ – 6 + 9λ + 9 + 16λ – 32 = 0 ⇒ 29λ – 29 = 0 ∴ λ = 1

∴ P ≡ (2, 5, 7) and hence $AP=\sqrt{1+36+16}=\sqrt{53}$

46. The equation of the plane through the points (2, 1, 0), (3, 2, – 2) and (3, 1, 7) is

A) 6x – 3y + 2z – 7 = 0 B) 3x – 2y + 6z – 27 = 0

C) 7x – 9y – z – 5 = 0 D) 2x – 3y + 4z – 27 = 0 **Ans. (C)**

Solution : (2, 1, 0) lies only on 7x – 9y – z – 4 = 0

Alternatively, use “determinant” = 0

47. The point of intersection of the line x + 1 = $\frac{y+3}{3}=\frac{-z+2}{2}$ with the plane 3x + 4y + 5z = 10 is

A) (2, 6, – 4) B) (– 2, 6, – 4) C) (2, 6, 4) D) (2, – 6, – 4) **Ans. (A)**

Solution : Any point on the line is (λ – 1, 3λ – 3, 2 – 2λ)

It will lie on the plane if 3(λ – 1) + 4(3λ – 3) + 5(2 – 2λ) = 10

i.e., 3λ – 3 + 12λ – 12 + 10 – 10λ = 10 ⇒ 5λ – 15 = 0 ∴ λ = 3

∴ The point is (2, 6, – 4)

Note : (2, 6, – 4) alone satisfies the plane.

48. If (2, 3, –1) is the foot of the perpendicular from (4, 2, 1) to a plane, then the equation of the plane is

A) 2x – y + 2z = 0 B) 2x – y + 2z + 1 = 0 C) 2x + y + 2z – 5 = 0 D) 2x + y + 2z – 1 = 0 **Ans. (B)**

Solution : The dr’s of the normal : 4 – 2, 2 – 3, 1 + 1 i.e., 2, –1, 2

Required **:** 2(x – 2) – 1 (y – 3) + 2 (z + 1) = 0 i.e., 2x – y + 2z = 1 = 0

Note : (2, 3, –1) satisfies equation in B alone.

49. $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=144$ and $|\vec{a}|=4$ then $|\vec{b}|$ is equal to

A) 8 B) 12 C) 4 D) 3 **Ans. (D)**

Solution : $144=4^2\,.|\vec{b}|^2 \Rightarrow |\vec{b}| = 3$

<u>Remark:</u> It is a repeated CET 18 question

50. If A and B are events such that $P(A)=\frac{1}{4},\ P(A/B)=\frac{1}{2}$ and $P(B/A)\ \frac{2}{3}$, then P(B) is

A) $\frac{2}{3}$ B) $\frac{1}{6}$ C) $\frac{1}{2}$ D) $\frac{1}{3}$ **Ans. (D)**

Solution : $\frac{P(A/B)}{P(B/A)}=\frac{P(A)}{P(B)}$ i.e., $\frac{\frac{1}{2}}{\frac{2}{3}}=\frac{\frac{1}{4}}{P(B)} \Rightarrow P(B)=\frac{\frac{1}{4}\cdot\frac{2}{3}}{\frac{1}{2}}=\frac{1}{3}$

51. A bag contains 2n + 1 coins. It is known that n of these coins have head on both sides whereas the other n + 1 coins are fair. One coin is selected at random and tossed. If the probability that toss results in heads is $\frac{31}{42}$, then the value of n is

A) 8 B) 5 C) 10 D) 6 **Ans. (C)**

Solution : A : coins having head on both sides

B : coins which are fair; E : Head

$P(E) = P(A \text{ and } E) + P(B \text{ and } E) = P(A) \times P(E/A) + P(B) \times P(E/B)$

$$= \frac{n}{2n+1}.1 + \frac{n+1}{2n+1}.\frac{1}{2} = \frac{31}{42} \Rightarrow n = 10$$

52. Let A = (x, y, z, u) and B = {a, b}. A function f : A → B is selected randomly. The probability that the function is an onto function is

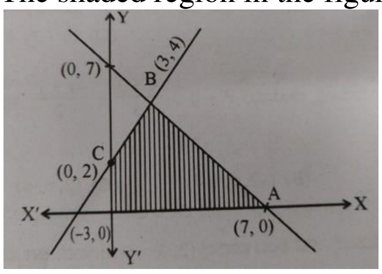
A) $\frac{5}{8}$ B) $\frac{7}{8}$ C) $\frac{1}{35}$ D) $\frac{1}{8}$ **Ans. (B)**

Solution : n(S) = Number of functions from A to B = $2^4 = 16$

n(A) = Number of onto function = $2^4 - 2 = 14$

$\therefore\ P(A) = \frac{14}{16} = \frac{7}{8}$

53. The shaded region in the figure given is the solution of which of the inequations ?



A) $x + y \geq 7,\ 2x - 3y + 6 \geq 0,\ x \geq 0,\ y \geq 0$ B) $x + y \leq 7,\ 2x - 3y + 6 \geq 0,\ x \geq 0,\ y \geq 0$

C) $x + y \leq 7,\ 2x - 3y + 6 \leq 0,\ x \geq 0,\ y \geq 0$ D) $x + y \geq 7,\ 2x - 3y + 6 \leq 0,\ x \geq 0,\ y \geq 0$ **Ans. (B)**

Solution : Equation of AB : $x + y = 7$; Equation of BC : $2x - 3y + 6 = 0$

$\therefore\ x + y \leq 7$ and $2x - 3y + 6 \geq 0$

54. If f(x) = ax + b, where a and b are integers, f(–1) = –5 and f(3) = 3 , then a and b are respectively

A) 0, 2 B) – 3, – 1 C) 2, 3 D) 2, – 3 **Ans. (D)**

Solution : $-a + b = -5$ and $3a + b = 3 \Rightarrow 4a = 8 \quad \therefore\ a = 2,\ b = -3$

55. The value of $e^{\log_{10} \tan 1° + \log_{10} \tan 2° + \log_{10} \tan 3° + ... + \log_{10} \tan 89°}$ is

A) $\frac{1}{e}$ B) 0 C) 1 D) 3 **Ans. (C)**

Solution : $e^{\log_{10} 1} = e^0 = 1$

56. The value of $\begin{vmatrix} \sin^2 14° & \sin^2 66° & \tan 135° \\ \sin^2 66° & \tan 135° & \sin^2 14° \\ \tan 135° & \sin^2 14° & \sin^2 66° \end{vmatrix}$ is

A) 1 B) –1 C) 2 D) 0 **Ans. (*)**

<u>Remark</u> : Error in the question; If it were $\sin^2 24°$ in the place of $\sin^2 14°$, then (D) 0 will be the correct answer.

57. The modulus of the complex number $\frac{(1+i)^2(1+3i)}{(2-6i)\,(2-2i)}$ is

A) $\frac{1}{\sqrt{2}}$ B) $\frac{4}{\sqrt{2}}$ C) $\frac{\sqrt{2}}{4}$ D) $\frac{2}{\sqrt{2}}$ **Ans. (C)**

Solution : $|z| = \frac{(\sqrt{2})^2\sqrt{10}}{\sqrt{40}\ \sqrt{8}} = \frac{2}{2.2\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

58. Given that a, b and x are real numbers and a < b, x < 0 then

A) $\frac{a}{x} < \frac{b}{x}$ B) $\frac{a}{x} > \frac{b}{x}$ C) $\frac{a}{x} \leq \frac{b}{x}$ D) $\frac{a}{x} \geq \frac{b}{x}$ **Ans. (B)**

59. Ten chairs are numbered as 1 to 10. Three women and two men wish to occupy one chair each. First the women choose the chairs marked 1 to 6, then the men choose the chairs from the remaining. The number of possible ways is

A) ${}^6C_3 \times {}^4P_2$ B) ${}^6C_3 \times {}^4C_2$ C) ${}^6P_3 \times {}^4C_2$ D) ${}^6P_3 \times {}^4P_2$ **Ans. (D)**

Solution : Required = ${}^6P_3 \times {}^4P_2$ or ${}^6C_3 \times {}^4C_2$ or ${}^6C_3 \times {}^7C_2$ or ${}^6P_3 \times {}^7P_2$

Remark: "Men choose the chairs form the remaining: - may mean that men can occupy the remaining 7 seats !?

If it is only choosing, then it is combination; otherwise permutation.

60. Which of the following is an empty set ?

A) $\{x : x^2 - 9 = 0, x \in R\}$ B) $\{x : x^2 - 1 = 0, x \in R\}$

C) $\{x : x^2 = x + 2, x \in R\}$ D) $\{x : x^2 + 1 = 0, x \in R\}$ **Ans. (D)**