MATHEMATICS
KARNATAKA CET - 2024

Version: **C-4**

KEY ANSWERS

1	C	16	C	31	C	46	C
2	B	17	D	32	B	47	B
3	D	18	A	33	A	48	C
4	A	19	C	34	A	49	B
5	D	20	A	35	B	50	*
6	B	21	C	36	D	51	C
7	C	22	A	37	C	52	A
8	A	23	D	38	A	53	B
9	C	24	C	39	C	54	C
10	B	25	B	40	C	55	B
11	D	26	B	41	D	56	B
12	D	27	C	42	B	57	A
13	A	28	B	43	C	58	B
14	C	29	B	44	C	59	D
15	B	30	C	45	C	60	B

* : None of the given options

1. The function x^X; $x > 0$, is strictly increasing at

(A) $\forall x \in R$ (B) $x < \frac{1}{e}$ C) $x > \frac{1}{e}$ (D) $x < 0$ **Ans: (C)**

Solution:

$$\frac{d}{dx}\left(x^x\right) = x^x\left(1 + \log x\right)$$

$$\uparrow \Rightarrow x^x\left(1 + \log x\right) > 0 \Rightarrow \log x > -1 \therefore x > \frac{1}{e}$$

2. The maximum volume of the right circular cone with slant height 6 units is

(A) $4\sqrt{3}\pi$ cubic units (B) $16\sqrt{3}\pi$ cubic units

(C) $3\sqrt{3}\pi$ cubic units (D) $6\sqrt{3}\pi$ it cubic units **Ans: (B)**

Solution: $V = \frac{1}{3}\pi r^2 h$; $l^2 = r^2 + h^2$

$$\therefore V = \frac{1}{3}\pi h\left(l^2 - h^2\right) = \frac{1}{3}\pi\left(l^2 h - h^3\right)$$

$$\frac{dV}{dh} = 0 \Rightarrow l^2 - 3h^2 = 0 \Rightarrow h = \frac{1}{\sqrt{3}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

$$\text{Max } V = \frac{1}{3}\pi\left(6^2 . 2\sqrt{3} - 24\sqrt{3}\right) = \frac{1}{3}\pi\left(48\sqrt{3}\right) = 16\pi\sqrt{3}$$

3. If $f(x) = x\, e^{x(1-x)}$ then f(x) is

(A) increasing in R (B) decreasing in R

(C) decreasing in $\left[-\frac{1}{2}, 1\right]$ (D) increasing in $\left[-\frac{1}{2}, 1\right]$ **Ans: (D)**

Solution: $f'(x) = e^{x(1-x)} + x \,.\, e^{x(1-x)} . (1 - 2x)$

$$= e^{x(1-x)}\left(1 + x - 2x^2\right)$$

$$f(x) \text{ is } \uparrow \Rightarrow 1 + x - 2x^2 > 0 \therefore x \in \left[-\frac{1}{2}, 1\right]$$

4. $\int \frac{\sin x}{3+4\cos^2 x} dx =$

(A) $-\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+C$ B) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{3}\right)+C$

C) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{3}\right)+C$ D) $-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{3}\right)+C$ **Ans: (A)**

Solution: $I = -\frac{1}{2}\int \frac{d(2\cos x)}{3+4\cos^2 x} = -\frac{1}{2}\cdot\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + C$

5. $\int_{-\pi}^{\pi}(1-x^2)\sin x \,.\cos^2 x\, dx =$

(A) $\pi - \frac{\pi^2}{3}$ (B) $2\pi - \pi^3$ (C) $\pi - \frac{\pi^3}{2}$ (D) 0 **Ans: (D)**

Solution: $(1 - x^2)\sin x \,.\, \cos^2 x$ is an odd function of x

$\therefore \int_{-\pi}^{\pi}\left(1-x^2\right)\sin x \,.\cos^2 x\, dx = 0$

6. $\int \frac{1}{x[6(\log x)^2+7\log x+2]} dx =$

A) $\frac{1}{2}\log\left|\frac{2\log x+1}{3\log x+2}\right| + C$ B) $\log\left|\frac{2\log x+1}{3\log x+2}\right| + C$

C) $\log\left|\frac{3\log x+2}{2\log x+1}\right| + C$ D) $\frac{1}{2}\log\left|\frac{3\log x+1}{3\log x+2}\right| + C$ **Ans: (B)**

Solution: Put $t = \log x$

Then $I = \int \frac{dt}{6t^2+7t+2}$; $6t^2 + 7t + 2 = 6t^2 + 4t + 3t + 2 = (3t+2)(2t+1)$

$= \int \frac{dt}{(3t+2)(2t+1)}$

$= \int\left(\frac{A}{(3t+2)} + \frac{B}{(2t+1)}\right)dt$; $A = \frac{1}{2\left(-\frac{2}{3}\right)+1} = -3$

$= \int\left(\frac{-3}{(3t+2)} + \frac{2}{(2t+1)}\right)dt$; $B = \frac{1}{3\left(-\frac{1}{2}\right)+2} = 2$

$= -\log(3t+2) + \log(2t+1)$

$= \log\frac{2t+1}{3t+2} + C$

7. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}} dx =$

A) $2x + \sin x + 2\sin 2x + C$ B) $x + 2\sin x + 2\sin 2x + C$

C) $x + 2\sin x + \sin 2x + C$ D) $2x + \sin x + \sin 2x + C$ **Ans: (C)**

Solution: $\frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}} = \frac{\sin\frac{5x}{2}\cos\frac{x}{2}}{\sin\frac{x}{2}\cos\frac{x}{2}} = \frac{\sin 3x + \sin 2x}{\sin x}$

$= \frac{3\sin x - 4\sin^3 x + 2\sin x\cos x}{\sin x} = 3 - 4\,.\,\frac{1}{2}(1-\cos 2x) + 2\cos x = 1 + 2\cos 2x + 2\cos x$

$\therefore I = x + \sin 2x + 2\sin x + C$

8. $\int_{1}^{5} (|x-3|+|1-x|)\, dx =$

A) 12 B) $\frac{5}{6}$ C) 21 D) 10 **Ans: (A)**

Solution: $I = \int_{1}^{5} |x-3|\, d(x-3) + \int_{1}^{5} |x-1|\, d(x-1)$

$= \frac{1}{2}(x-3)|x-3|\Big|_{1}^{5} + \frac{1}{2}(x-1)|x-1|\Big|_{1}^{5} = \frac{1}{2}(4+4) + \frac{1}{2}(16+0) = 4+8 = 12$

9. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+......+\frac{1}{(5n)}\right) =$

A) $\frac{\pi}{4}$ B) $\tan^{-1} 3$ 3) $\tan^{-1} 2$ D) $\frac{\pi}{2}$ **Ans: (C)**

Solution: $L = \lim_{n\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2}$; $\frac{1}{5n} = \frac{n}{n^2+(2n)^2}$

$= \underset{n\to\infty}{\text{Lt}}\sum_{r=1}^{2n}\frac{1}{n}\cdot\frac{1}{1+\left(\frac{r}{n}\right)^2} = \int_0^2 \frac{1}{1+x^2}dx = \tan^{-1}x\Big|_0^2 = \tan^{-1}2$

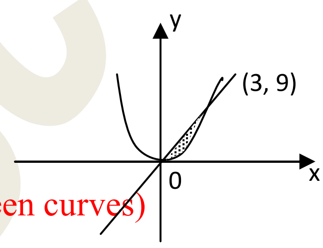
<u>**Remark**</u>: This question is from the DELETED syllabus (integration as summation)

10. The area of the region bounded by the line $y = 3x$ and the curve $y = x^2$ in sq. units is

A) 10 B) $\frac{9}{2}$ C) 9 D) 5 **Ans: (B)**

Solution: $A = \int_0^3 (3x - x^2)dx$

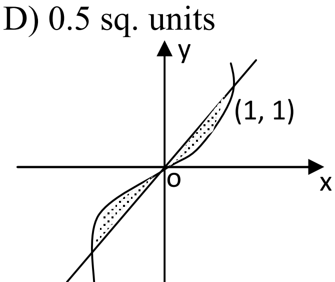
$= \left(3\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^3 = \frac{27}{2} - \frac{27}{3} = 27 \cdot \frac{1}{6} = \frac{9}{2}$



<u>**Remark**</u>: This is also from the DELETED syllabus (area between curves)

11. The area of the region bounded by the line $y = x$ and the curve $y = x^3$ is

A) 0.2 sq. units B) 0.3 sq. units C) 0.4 sq. units D) 0.5 sq. units **Ans: (D)**



Solution: $A = 2\int_0^1 (x - x^3)dx$

$A = 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{4}\right] = \frac{1}{2} = 0.5$

<u>**Remark**</u>: This is also from the DELETED syllabus (area between curves)

12. The solution of $e^{\frac{dy}{dx}} = x + 1$, $y(0) = 3$ is

(A) $y - 2 = x \log x - x$ (B) $y - x - 3 = x \log x$

(C) $y - x - 3 = (x + 1) \log (x + 1)$ (D) $y + x - 3 = (x + 1)\log(x + 1)$ **Ans: (D)**

Solution: $e^{\frac{dy}{dx}} = x+1 \Rightarrow \frac{dy}{dx} = \log(x+1)$

$\therefore y = \int \log(x+1)\, d(x+1) = \log(x+1)\cdot(x+1) - \int \frac{1}{x+1}\cdot(x+1)dx$

$= (x+1)\log(x+1) - x + c$

$y(0) = 3 \Rightarrow 3 = 0 - 0 + c \Rightarrow c = 3$

$\therefore y = (x+1)\log(x+1) - x + 3$ i.e. $y + x - 3 = (x + 1) \log (x + 1)$

13. The family of curves whose x and y intercepts of a tangent at any point are respectively double the x and y coordinates of that point is

(A) $xy = C$ (B) $x^2 + y^2 = C$ (C) $x^2 - y^2 = C$ (D) $\frac{y}{x} = C$ **Ans: (A)**

Solution: Equation of tangent at (x_1, y_1) ; $y - y_1 = m(x - x_1)$; $m = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}$

By data $(2x_1, 0)$ and $(0, 2y_1)$ lie on it

$\Rightarrow -y_1 = mx_1 \Rightarrow m = \frac{-y_1}{x_1}\left[\text{also}, y_1 = m(-x_1)\right]$

Thus, $\frac{dy}{dx}$ at (x, y) is $\frac{-y}{x}$ i.e. $\frac{dy}{dx} = \frac{-y}{x}$

$\Rightarrow x\,dy + y\,dx = 0$ i.e. $d(xy) = 0 \therefore xy = C$

Remark: From DELETED syllabus (tangent and normal)

14. The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a Δ ABC. The length of the median through A is

(A) $\sqrt{18}$ (B) $\sqrt{72}$ (C) $\sqrt{33}$ (D) $\sqrt{288}$ **Ans: (C)**

Solution: $\overrightarrow{AD} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{AC}\right) = \frac{1}{2}(8i - 2j + 8k) = 4i - j + 4k$

$\therefore AD = \left|\overrightarrow{AD}\right| = \sqrt{16 + 1 + 16} = \sqrt{33}$

15. The volume of the parallelopiped whose co-terminous edges are $\hat{j} + \hat{k}$, $\hat{i} + \hat{k}$ and $\hat{i} + \hat{j}$ is

(A) 6 cu. units (B) 2 cu. Units (C) 4 cu. units (D) 3 cu. Units **Ans: (B)**

Solution: Required $= [i + j \quad j + k \quad k + i]$

$= 2[i \ j \ k] =$ 2 cu units

Remark: From DELETED syllabus (Scalar triple product of vectors)

16. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if

(A) $\theta = \frac{\pi}{4}$ (B) $\theta = \frac{\pi}{3}$ (C) $\theta = \frac{2\pi}{3}$ (D) $\theta = \frac{\pi}{2}$ **Ans: (C)**

Solution: $|\vec{a} + \vec{b}| = 2\cos\frac{\theta}{2} \Rightarrow \cos\frac{\theta}{2} = \frac{1}{2} \therefore \theta = \frac{2\pi}{3}$

17. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors and p, q, r are vectors defined by

$\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, then $(\vec{a} + \vec{b}).\vec{p} + (\vec{b} + \vec{c}).\vec{q} + (\vec{c} + \vec{a}).\vec{r}$ is

(A) 0 (B) 1 (C) 2 (D) 3 **Ans: (D)**

Solution: $\vec{a}.\vec{p} = \vec{q}.\vec{b} = \vec{r}.\vec{c} = 1$

$\vec{b}.\vec{p} = \vec{c}.\vec{q} = \vec{a}.\vec{r} = 0$

$\therefore$ G. E. $= 3$

Remark: From DELETED syllabus (Scalar triple product of vectors)

18. If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then k is equal to

(A) $-\frac{10}{7}$ (B) $-\frac{7}{10}$ (C) -10 (D) -7 **Ans: (A)**

Solution: $(-3)(3k) + (2k)(1) + 2(-5) = 0$

$\therefore 7k = -10 \therefore k = \frac{-10}{7}$

19. The distance between the two planes 2x + 3y + 4z = 4 and 4x + 6y + 8z = 12 is

(A) 2 units (B) 8 units (C) $\frac{2}{\sqrt{29}}$ units (D) 4 units **Ans: (C)**

Solution: Required = $\frac{6-4}{\sqrt{4+9+16}} = \frac{2}{\sqrt{29}}$

Remark: From DELETED syllabus (The plane)

20. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{4-z}{-5}$ and the plane 2x – 2y + z = 5 is

(A) $\frac{1}{5\sqrt{2}}$ (B) $\frac{2}{5\sqrt{2}}$ (C) $\frac{3}{50}$ (D) $\frac{3}{\sqrt{50}}$ **Ans: (A)**

Solution: $\sin\theta = \frac{|ab+bm+cn|}{\sqrt{\sum a^2}\sqrt{\sum l^2}} = \frac{|6-8+5|}{\sqrt{9+16+25}\sqrt{4+4+1}} = \frac{3}{\sqrt{50}.3} = \frac{1}{\sqrt{50}} = \frac{1}{5\sqrt{2}}$

Remark: From DELETED syllabus (The plane)

21. The equation xy = 0 in three-dimensional space represents

(A) a pair of straight lines (B) a plane

(C) a pair of planes at right angles (D) a pair of parallel planes **Ans: (C)**

Solution: xy = 0 ⇒ x = 0 and y = 0 i.e. y – z plane and z – x plane ; ⊥ planes .

22. The plane containing the point (3, 2, 0) and the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is

(A) x – y + z = 1 (B) x + y + z = 5 (C) x + 2y – z = 1 (D) 2x – y + z = 5 **Ans: (A)**

Solution: $\begin{vmatrix} x-3 & y-6 & z-4 \\ 0 & -4 & -4 \\ 1 & 5 & 4 \end{vmatrix} = 0 \Rightarrow x - y + z = 1$

Aliter: The point (3, 2, 0) satisfies (A) and (D)

But $a_1a_2 + b_1b_2 + c_1c_2 = 0$, only for (A) ; (1) (1) + 5(– 1) + (1) (4) = 0

Remark: From DELETED syllabus (The plane)

23. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).
Let z = 4x + 6y be the objective function. The minimum value of z occurs at

(A) Only (0, 2) (B) Only (3, 0)

(C) The mid-point of the line segment joining the points (0, 2) and (3, 0)

(D) Any point on the line segment joining the points (0, 2) and (3, 0) **Ans: (D)**

Solution: Z(0, 2) = 12; Z (3, 0) = 12 ; other Z values are obviously more than 12

Z_{min} occurs at (0, 2) and (3, 0) and hence at every point on the line segment joining them.

24. A die is thrown 10 times. The probability that an odd number will come up at least once is

(A) $\frac{11}{1024}$ (B) $\frac{1013}{1024}$ (C) $\frac{1023}{1024}$ (D) $\frac{1}{1024}$ **Ans: (C)**

Solution: P(atleast one) = 1 – P(none)

$$= 1 - \left(\frac{1}{2}\right)^{10} = \frac{1023}{1024}$$

Remark: From DELETED syllabus (Binomial Distribution)

25. A random variable X has the following probability distribution

X	0	1	2
P(X)	$\frac{25}{36}$	k	$\frac{1}{36}$

If the mean of the random variable X is $\frac{1}{3}$, then the variance is

(A) $\frac{1}{18}$ (B) $\frac{5}{18}$ (C) $\frac{7}{18}$ (D) $\frac{11}{18}$ **Ans: (B)**

Solution: $\sum p(x) = 1 \Rightarrow \frac{25}{36} + k + \frac{1}{36} = 1 \Rightarrow k = \frac{10}{36} = \frac{5}{18}$

$\sigma^2 = \sum px^2 - (\bar{x})^2 = \left(0 + \frac{5}{18} \cdot 1^2 + \frac{1}{36} \cdot 4\right) - \left(\frac{1}{3}\right)^2 = \frac{5}{18}$

Remark: From DELETED syllabus (Probability Distribution)

26. If a random variable X follows the binomial distribution with parameters n = 5, p and P(X = 2) = 9P(X = 3), then p is equal to

(A) 10 (B) $\frac{1}{10}$ (C) 5 (D) $\frac{1}{5}$ **Ans: (B**)

Solution: $P(x) = {}^nC_x\, p^x q^{n-x}$; $p + q = 1$

$p(x = 2) = 9p(x = 3) \Rightarrow {}^5C_2 p^2 q^3 = 9.{}^5C_3\, p^3 q^2 \Rightarrow q = 9p$ i.e. $1 - p = 9p \Rightarrow p = \frac{1}{10}$

Remark: From DELETED syllabus (Binomial Distribution)

27. Two finite sets have in and n elements respectively. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n respectively are

(A) 7, 6 (B) 5, 1 (C) 6, 3 (D) 8, 7 **Ans: (C**)

Solution: $2^m - 2^n = 56 = 8 \times 7 = 2^6 - 2^3 \Rightarrow m = 6, n = 3$

28. If $[x]^2 - 5[x] + 6 = 0$, where [x] denotes the greatest integer function, then

(A) $x \in [3, 4]$ (B) $x \in [2, 4)$ (C) $x \in [2, 3]$ (D) $x \in (2, 3]$ **Ans: (B**)

Solution: [x] = 2 or 3 $\therefore\ x \in [2, 4)$

29. If in two circles, arcs of the same length subtend angles 30° and 78° at the centre, then the ratio of their radii is

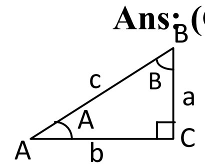
(A) $\frac{5}{13}$ (B) $\frac{13}{5}$ (C) $\frac{13}{4}$ (D) $\frac{4}{13}$ **Ans: (B**)

Solution: $\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{78}{30} = \frac{13}{5}$ (because it is a ratio)

30. If ΔABC is right angled at C, then the value of tan A + tan B is

(A) a + b (B) $\frac{a^2}{bc}$ (C) $\frac{c^2}{ab}$ (D) $\frac{c^2}{ac}$ **Ans: (C)**

Solution: $\tan A + \tan B = \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab} = \frac{c^2}{ab}$



31. The real value of 'α' for which $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely real is

(A) $(n + 1)\frac{\pi}{2}$, $n \in N$ (B) $(2n + 1)\frac{\pi}{2}$, $n \in N$ (C) $n\pi$, $n \in N$ (D) $(2n - 1)\frac{\pi}{2}$, $n \in N$ **Ans. (C)**

Solution: $Z = \frac{(1 - i\sin\alpha)(1 - 2i\sin\alpha)}{1 + 4\sin^2\alpha} = \frac{(\quad) - 3i\sin\alpha}{(\ )}$

Z is purely real $\Rightarrow \frac{-3\sin\alpha}{1 + 4\sin^2\alpha} = 0 \Rightarrow \sin\alpha = 0 \quad \therefore\ \alpha = n\pi$

Aliter: When α = 0, Z is real

And α = 0 is included only in (C)

32. The length of a rectangle is five times the breadth. If the minimum perimeter of the rectangle is 180 cm, then

(A) Breadth ≤ 15 cm (B) Breadth ≥ 15 cm (C) Length ≤ 15 cm (D) Length = 15 cm

Ans. (B)

Solution : $l = 5b$

$P = 2(5b + b) = 12b \geq 180 \Rightarrow b \geq 15$

33. The value of ${}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3 + {}^{45}C_4$ is

(A) ${}^{50}C_4$ (B) ${}^{50}C_3$ (C) ${}^{50}C_2$ (D) ${}^{50}C_1$ **Ans. (A)**

Solution: Use ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

${}^{45}C_3 + {}^{45}C_4 = {}^{46}C_4$; ${}^{46}C_4 + {}^{46}C_3 = {}^{47}C_4$; ${}^{47}C_3 + {}^{47}C_4 = {}^{48}C_4$ etc

34. In the expansion of $(1 + x)^n$

$\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + ... + n\frac{C_n}{C_{n-1}}$ is equal to

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n}{2}$ (C) $\frac{n+1}{2}$ (D) $3n(n + 1)$ **Ans: (A)**

Solution: $\frac{C_r}{C_{r-1}} = \frac{{}^{n}C_r}{{}^{n}C_{r-1}} = \frac{n-r+1}{r}$

$\therefore \text{G.E.} = \sum_{r=1}^{n} r \cdot \frac{(n-r+1)}{r} = \sum_{r=1}^{n}(n-r+1) = n + (n-1) + ...+ 1 = \frac{n(n+1)}{2}$

Aliter: Put n = 1: Then $\text{G.E.} = \frac{{}^{1}C_1}{{}^{1}C_0} = 1$

Option (A) : $= \frac{{}^{1}C_{1+1}}{2} = 1$ (C) $\frac{1+1}{2} = 1$ satisfy

Put n = 2: Then $\text{G.E.} = \frac{{}^{2}C_1}{{}^{2}C_0} + 2 \cdot \frac{{}^{2}C_2}{{}^{2}C_1} = \frac{2}{1} + 2 \cdot \frac{1}{2} = 3$; Only A satisfies

Remark: From DELETED Syllabus (Binomial Coefficients)

35. If S_n stands for sum to n-terms of a G.P. with 'a' as the first term and 'r' as the common ratio then $S_n : S_{2n}$ is

(A) $r^n + 1$ (B) $\frac{1}{r^n + 1}$ (C) $r^n - 1$ (D) $\frac{1}{r^n - 1}$ **Ans. (B)**

Solution: $\frac{S_n}{S_{2n}} = \frac{\frac{a(r^n - 1)}{r-1}}{\frac{a(r^{2n} - 1)}{r-1}} = \frac{r^n - 1}{(r^n + 1)\ (r^n - 1)} = \frac{1}{r^n + 1}$

Aliter: Put n = 1

Then $\frac{S_n}{S_{2n}} = \frac{S_1}{S_2} = \frac{a}{a + ar} = \frac{1}{1 + r}$; (B) alone gives $\frac{1}{r+1}$ when n = 1

36. If A.M. and G.M. of roots of a quadratic equation are 5 and 4 respectively, then the quadratic equation is

(A) $x^2 - 10x - 16 = 0$ (B) $x^2 + 10x + 16 = 0$

(C) $x^2 + 10x - 16 = 0$ (D) $x^2 - 10x + 16 = 0$ **Ans: (D)**

Solution: $x^2 - 2Ax + G^2 = 0$ i.e. $x^2 - 10x + 16 = 0$

37. The angle between the line x + y = 3 and the line joining the points (1, 1) and (–3, 4) is

(A) $\tan^{-1}(7)$ (B) $\tan^{-1}\left(-\frac{1}{7}\right)$ (C) $\tan^{-1}\left(\frac{1}{7}\right)$ (D) $\tan^{-1}\left(\frac{2}{7}\right)$ **Ans. (C)**

Solution: $m_1 = -1$; $m_2 = \frac{4-1}{-3-1} = -\frac{3}{4}$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-1+\frac{3}{4}}{1+\frac{3}{4}}\right| = \left|-\frac{1}{7}\right| = \frac{1}{7} \quad \therefore \theta = \tan^{-1}\frac{1}{7}$$

Note: If we apply $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$ gives $\tan\theta = -\frac{1}{7}$

$\therefore \theta = \pi - \tan^{-1}\frac{1}{7}$ is the obtuse angle between the lines

$\therefore \theta = \tan^{-1}\left(-\frac{1}{7}\right) \in \left(-\frac{\pi}{2}, 0\right)$ can't be the correct option

38. The equation of parabola whose focus is (6, 0) and directrix is x = –6 is

(A) $y^2 = 24x$ (B) $y^2 = -24x$ (C) $x^2 = 24y$ (D) $x^2 = -24y$ **Ans. (A)**

Solution: Standard form I : $y^2 = 4ax$; a = 6 $\therefore$ $y^2 = 24x$

39. $\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$ is equal to

(A) 2 (B) $\sqrt{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$ **Ans: (C)**

Solution: $L = \lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = \lim_{x \to \frac{\pi}{4}} \frac{-\sqrt{2}\sin x}{-\operatorname{cosec}^2 x} = \frac{1}{2}$

40. The negation of the statement

"For every real number x; $x^2 + 5$ is positive" is

(A) For every real number x; $x^2 + 5$ is not positive.

(B) For every real number x; $x^2 + 5$ is negative.

(C) There exists at least one real number x such that $x^2 + 5$ is not positive.

(D) There exists at least one real number x such that $x^2 + 5$ is positive. **Ans. (C)**

Remark: From DELETED Syllabus (Mathematical Reasoning)

41. Let a, b, c, d and e be the observations with mean m and standard deviation S. The standard deviation of the observations a + k, b + k, c + k, d + k and e + k *is*

(A) kS (B) S + k (C) $\frac{S}{k}$ (D) S **Ans. (D)**

Solution: Standard deviation remains the same if we increase or decrease the items by a constant

42. Let f : R → R be given by f(x) = tan x. Then $f^{-1}(1)$ is

(A) $\frac{\pi}{4}$ (B) $\left\{n\pi + \frac{\pi}{4} : n \in Z\right\}$ (C) $\frac{\pi}{3}$ (D) $\left\{n\pi + \frac{\pi}{3} : n \in Z\right\}$ **Ans: (B)**

Solution: Let $a \in f^{-1}(1) \Rightarrow f(a) = 1$ i.e. $\tan a = 1 \Rightarrow a = n\pi + \frac{\pi}{4}$

Remark: f^{-1} (b) i.e. inverse of an element **:** From DELETED syllabus and also general solution of trig. equations. (Deleted)

43. Let f : R → R be defined by $f(x) = x^2 + 1$. Then the pre images of 17 and – 3 respectively are

(A) ϕ, {4, – 4} (B) {3, – 3}, ϕ (C) {4, – 4}, ϕ (D) {4, – 4}, {2, – 2} **Ans: (C)**

Solution: $x^2 + 1 = 17 \Rightarrow x = \pm 4$; $x^2 + 1 \neq -3$

$\therefore$ (C) is the correct option

Remark: Inverse of an element is from the DELETED syllabus

44. Let (gof) (x) = sin x and (fog) (x) = $(\sin \sqrt{x})^2$. Then

(A) $f(x) = \sin^2 x$, $g(x) = x$ (B) $f(x) = \sin \sqrt{x}$, $g(x) = \sqrt{x}$

(C) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$ (D) $f(x) = \sin \sqrt{x}$, $g(x) = x^2$ **Ans. (C)**

Solution: (gof) (x) = g(f(x)) = sin x = $\sqrt{\sin^2 x}$, assuming, sin x > 0

$\therefore g(x) = \sqrt{x},\ f(x) = \sin^2 x$ and $f(g(x)) = \sin^2 g(x) = (\sin\sqrt{x})^2$

45. Let A = {2, 3, 4, 5............16, 17, 18}. Let R be the relation on the set A of ordered pairs of positive integers defined by (a, b) R (c, d) if and only if ad = bc for all (a, b), (c, d) in A × A. Then the number of ordered pairs of the equivalence class of (3, 2) is

(A) 4 (B) 5 (C) 6 (D) 7 **Ans. (C)**

Solution: The given relation is the equality relation on "fractions" : $\frac{a}{b} = \frac{c}{d}$ if equivalence class of (3, 2)

$= \left\{(a, b) : \frac{a}{b} = \frac{3}{2}\right\} : \frac{3}{2} = \frac{6}{4} = \frac{9}{6} = \frac{12}{8} = \frac{15}{10} = \frac{18}{12}$

= {(3, 2), (6, 4), (9, 4), (12, 8), (15, 10), (18, 12)}

Number of ordered points is 6

46. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then x (y + z) + y (z + x) + z (x + y) equals to

(A) 0 (B) 1 (C) 6 (D) 12 **Ans. (C)**

Solution: $0 \le \cos^{-1} x, \cos^{-1} y, \cos^{-1} z \le \pi$ and $\cos^{-1}(-1) = \pi$

$\therefore$ By data, x = –1 = y = z $\therefore$ G.E. = 2 + 2 + 2 = 6

47. If $2 \sin^{-1} x - 3 \cos^{-1} x = 4$, $x \in [-1, 1]$ then $2 \sin^{-1} x + 3 \cos^{-1} x$ is equal to

(A) $\frac{4-6\pi}{5}$ (B) $\frac{6\pi-4}{5}$ (C) $\frac{3\pi}{2}$ (D) 0 **Ans. (B)**

Solution: $2 \sin^{-1} x + 3 \cos^{-1} x = 2 (\sin^{-1} x + \cos^{-1} x) + \cos^{-1} x = \pi + \cos^{-1} x$

Now, $2 \sin^{-1} x - 3 \cos^{-1} x = 4 \Rightarrow 2\left(\frac{\pi}{2} - \cos^{-1} x\right) - 3\cos^{-1} x = 4$

$\therefore 5 \cos^{-1} x = \pi - 4 \Rightarrow \cos^{-1} x = \frac{1}{5}(\pi - 4)$

$\therefore$ Required $= \pi + \frac{1}{5}(\pi - 4) = \frac{6\pi - 4}{5}$

Remark: From DELETED Syllabus (Properties of Inv. Trig. Functions)

48. If A is a square matrix such that $A^2 = A$, then $(I + A)^3$ is equal to

(A) 7A – I (B) 7A (C) 7A + I (D) I – 7A **Ans. (C)**

Solution: $A^2 = A \Rightarrow A^3 = A^2 = A$

$(I + A)^3 = I + 3A + 3A^2 + A^3 = I + 3A + 3A + A = 7A + I$

49. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{10} is equal to

(A) 2^8 A (B) 2^9 A (C) 2^{10} A (D) 2^{11} A **Ans. (B)**

Solution: Clearly, $A^2 = 2A$; $A^3 = 2A^2 = 2^2 A$

$|||^{ly}$ proceeding, $A^4 = 2^3 A$, ….., $A^{10} = 2^9 A$

50. If $f(x) = \begin{vmatrix} x-3 & 2x^2-18 & 2x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix}$, then f(1) . f(3) + f(3) . f(5) + f(5) . f(1) is

(A) –1 (B) 0 (C) 1 (D) 2 **Ans. (*)**

Solution : f(5) = 0 but f(1) ≠ 0, f(3) ≠ 0 (check)

$\therefore$ G.E. = f(1) . f(3) + 0 + 0 , |f(1) f(3)| > 2

$\therefore$ None of the given options is the answer

Suppose $a_{13} = 3x^3 - 81$, then f(3) = 0 and hence Δ = 0

Remark: It is a FAULTY question from DELETED Syllabus (Properties of Determinants)

51. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to

(A) 4 (B) 5 (C) 11 (D) 0 **Ans. (C)**

Solution: $|P| = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6)$

i.e., $|P| = |A|^2 = 16 = 0 + 2\alpha - 6 \Rightarrow \alpha = 11$

52. If $A = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$ and $B = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$, then $\frac{dB}{dx}$ is

(A) 3A (B) – 3B (C) 3B + 1 (D) 1 – 3A **Ans. (A)**

Solution: $A = x^2 - 1$; $B = x(x^2 - 1) - 1(x - 1) + 1(1 - x) = x^3 - 3x + 2$

$\therefore \frac{dB}{dx} = 3(x^2 - 1) = 3A$

53. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$. Then $\lim_{x \to 0} \frac{f(x)}{x^2} =$

(A) –1 (B) 0 (C) 3 (D) 2 **Ans. (B)**

Solution: $\frac{f(x)}{x^2} = \frac{1}{x^2}\begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix} = \begin{vmatrix} \cos x & x & 1 \\ \frac{2\sin x}{x} & 1 & 2 \\ \frac{\sin x}{x} & 1 & 1 \end{vmatrix}$

$\lim_{x \to 0} \frac{f(x)}{x^2} = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 1(1-2) + 1(2-1) = 0$

Aliter: Without using properties of determinants:

$f(x) = \cos x \,.\, (x^2 - 2x^2) - x(2x \sin x - 2x \sin x) + (2x \sin x - x \sin x) = -x^2 \cos x + x \sin x$

$\lim_{x \to 0} \frac{f(x)}{x} = \lim_{x \to 0}\left(-\cos x + \frac{\sin x}{x}\right) = -1 + 1 = 0$

54. Which one of the following observations is correct for the features of logarithm function to any base $b > 1$?

(A) The domain of the logarithm function is R, the set of real numbers.

(B) The range of the logarithm function is R^+, the set of all positive real numbers.

(C) The point (1, 0) is always on the graph of the logarithm function.

(D) The graph of the logarithm function is decreasing as we move from left to right. **Ans. (C)**

Solution: y = log x always satisfies (1, 0)

55. The function $f(x) = |\cos x|$ is

(A) everywhere continuous and differentiable

(B) everywhere confinuous but not differentiable at odd multiples of $\frac{\pi}{2}$

(C) neither continuous nor differentiable at $(2n + 1)\frac{\pi}{2}, n \in Z$

(D) not differentiable everywhere **Ans. (B)**

Solution: We know that $|f(x)|$ is continuous whenever f(x) is continuous but not differentiable whenever $f(x) = 0$

Here $f(x) = \cos x$ and hence $|\cos x|$ is continuous but not differentiable whenever $\cos x = 0$

i.e., when x is an odd multiple of $\frac{\pi}{2}$

56. If $y = 2x^{3x}$, then $\frac{dy}{dx}$ at x = 1 is

(A) 2 (B) 6 (C) 3 (D) 1 **Ans. (B)**

Solution: $y = 2.x^{3x} \Rightarrow \frac{dy}{dx} = 2\left[x^{3x}.\ \frac{d}{dx}(3x\ \ \log\ \ x)\right]\ =\ 2.x^{3x}\left[3.\log x + 3x.\frac{1}{x}\right]$

$\therefore$ At $x = 1, \frac{dy}{dx} = 2.1.(3\log 1 + 3) = 6$

57. Let the function satisfy the equation f(x + y) = f(x) f(y) for all x, y $\in$ R, where f(0) $\neq$ 0. If f(5) = 3 and $f'(0) = 2$, then f '(5) is

(A) 6 (B) 0 (C) 5 (D) – 6 **Ans. (A)**

Solution: f(x + y) = f(x) f(y). Then f(x) can be a^x ; f(5) = a^5 = 3

Then $f'(x) = a^x \log a$; $f'(0) = 2 \Rightarrow \log a = 2$

$f'(5) = a^5 \log a = 3(2) = 6$

Aliter: f(x + 5) = f(x) . f(5) $\Rightarrow$ $f'(x + 5) = f'(x) . f(5)$

Put x = 0 : $f'(5) = f'(0) . f(5) = 2 \times 3 = 6$

Third method:

$$f'(5) = \lim_{h\to 0}\frac{f(5+h) - f(5)}{h};\ \ f(0) = 1$$

$$= \lim_{h\to 0}\ \frac{f(5).f(h) - f(5)}{h} = \lim_{h\to 0} f(5).\frac{f(h) - 1}{h} = f(5).f'(0) = 6$$

58. The value of C in (0, 2) satisfying the mean value theorem for the function $f(x) = x(x - 1)^2$, x $\in$ [0, 2] is equal to

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$ **Ans. (B)**

Solution: $f'(x) = 1 . (x - 1)^2 + x . 2(x - 1) = x^2 - 2x + 1 + 2x^2 - 2x = 3x^2 - 4x + 1$

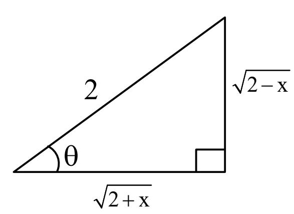
$$f'(c) = \frac{f(2) - f(0)}{2 - 0} \Rightarrow 3c^2 - 4c + 1 = \frac{2-0}{2-0} = 1 \Rightarrow c(3c - 4) = 0 \Rightarrow c = \frac{4}{3} \in (0, 2)$$

Remark: This is from DELETED Syllabus.

59. $\frac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\frac{2+x}{2-x}}\right)\right]$ is

(A) $-\frac{3}{4}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$ **Ans. (D)**

Solution: $\cot^{-1}\sqrt{\frac{2+x}{2-x}}\ =\ \theta \Rightarrow \cot\theta = \frac{\sqrt{2+x}}{\sqrt{2-x}}$



$$y = \cos^2\left[\cot^{-1}\sqrt{\frac{2+x}{2-x}}\right]\ =\ \cos^2\theta = \frac{2+x}{4}\quad \therefore \frac{dy}{dx} = \frac{1}{4}$$

Aliter: put x = 2 cos θ. Then $\frac{2+x}{2-x} = \frac{2(1+\cos\theta)}{2(1-\cos\theta)} = \cot^2\frac{\theta}{2}$

$$\therefore\ \ y = \cos^2\left(\cot^{-1}\cot\frac{\theta}{2}\right)\ =\ \cos^2\frac{\theta}{2} = \frac{1}{2}(1+\cos\theta) = \frac{1}{2}\left(1+\frac{x}{2}\right)\ \Rightarrow\ y' = \frac{1}{4}$$

60. For the function $f(x) = x^3 - 6x^2 + 12x - 3$; x = 2 is

(A) a point of minimum (B) a point of inflexion

(C) not a critical point (D) a point of maximum **Ans. (B)**

Solution: $f'(x) = 3x^2 - 12x + 12$; $f''(x) = 6x - 12$; $f'''(x) = 6 \neq 0$

$f'(2) = 0 = f''(2)$ $\therefore$ x = 2 is a point of inflexion.